Cambridge Finance Seminar
Cambridge 7 March 2008

Systematic Investment

M A H Dempster

Statistical Laboratory
University of Cambridge
&
Cambridge Systems Associates Limited

mahd2@cam.ac.uk www-cfr.jbs.cam.ac.uk

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Outline

- Introduction
- Systematic Investment Strategy Survey
- Systematic Portfolio Construction Overview
- Long/Short Equity Strategies for Alpha
- Institutional ALM
- Individual ALM
- Conclusion

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Issues

- Trading, investing or asset liability management?
- Institutional or individual investment?
- Hedge fund, active or passive mutual fund, DB or DC pension fund or insurance fund?
- Strategic, tactical or operational portfolio construction?
- Short, medium or long term horizon?
- Traditional securities, alternatives, commodities, overlays or derivatives?

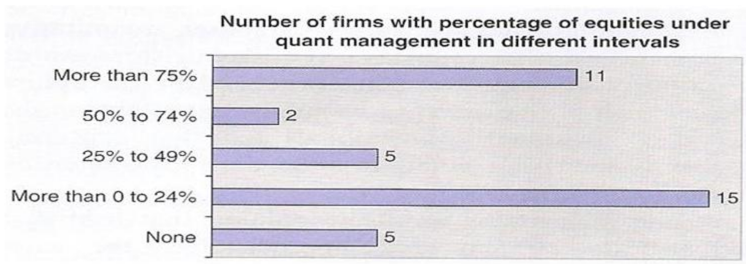
© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Systematic Investment Strategy Survey

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Systematic Investment Strategies Acceptance

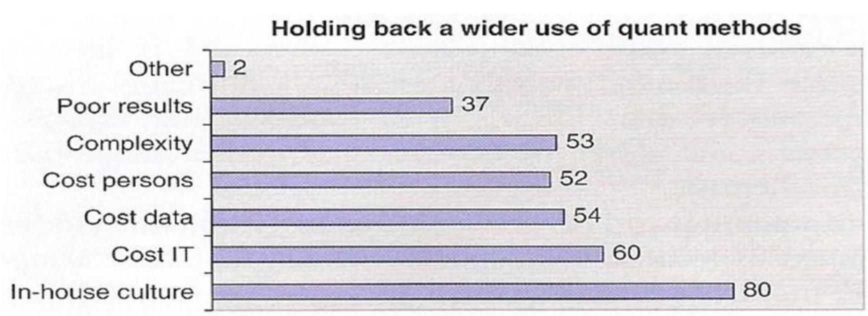
- Survey of equity managers' acceptance of systematic investment strategies
  - 38 firms in Europe and North America with a total of € 3.3 trillion in equities under management Fabozzi *et al* (2007) in special issue of *Quantitative Finance* on strategic investment



Distribution of the percentage of equities under quant management

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

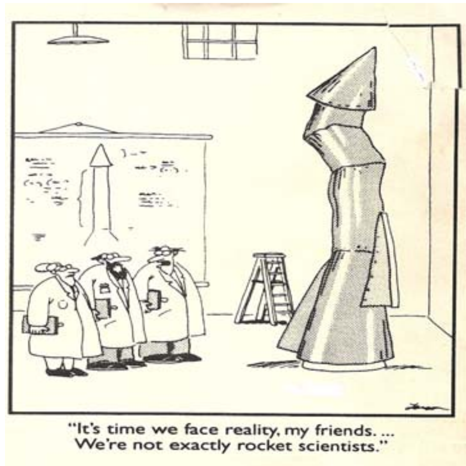
Factors Delaying Wider Use



Score attributed to each factor holding back a wider use of quant methods

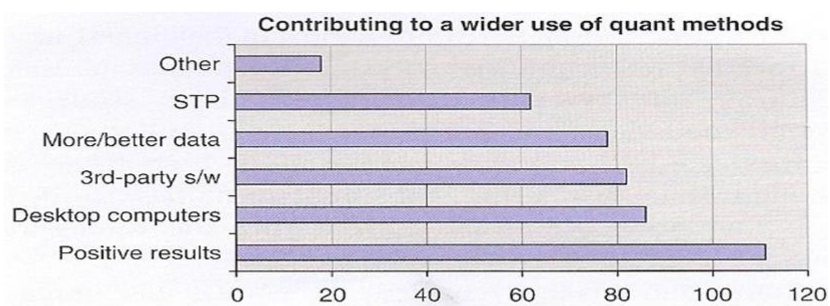
© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

And Here is the Reason Why!



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

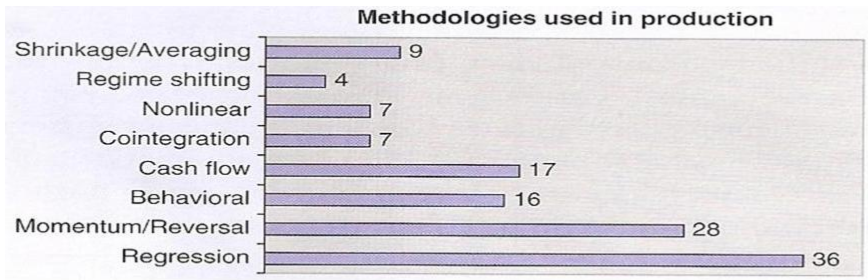
Factors Contributing to Wider Use



Score attributed to each factor contributing to a wider use of quant methods

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

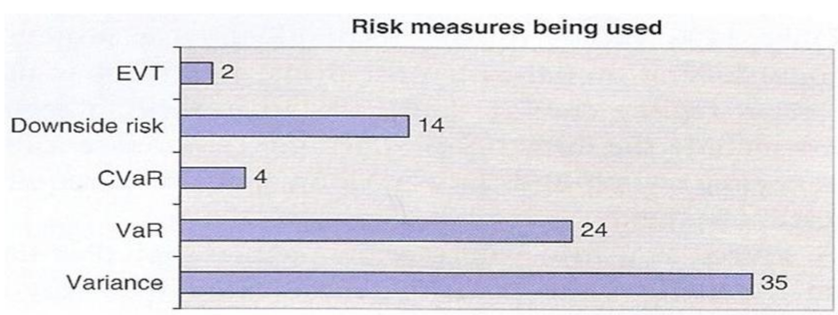
Quantitative Methodologies Used



Distribution of modelling methodologies among participants

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Risk Measures Used

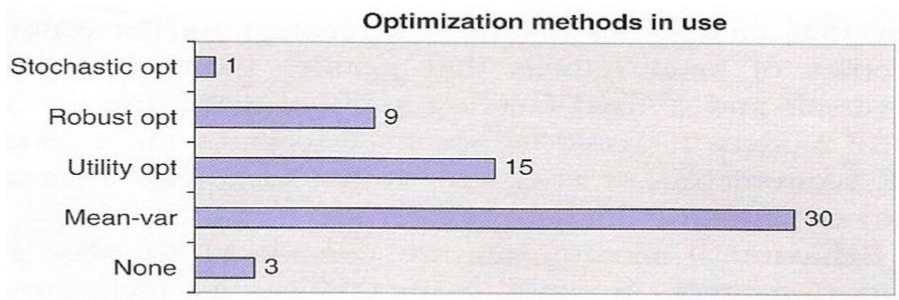


Distribution of risk measures adopted by participants

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

11

Optimization Methods for Portfolio Construction



Distribution of optimization methods adopted by participants

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

12

Findings Summary

- The authors remark that despite a slow down in the acceptance of systematic techniques after well documented disasters with their use – such as in 1998 and 2007? – progress is inexorable!
- Due to the influence of hedge funds on the industry employment of systematic investment strategies is moving from tactical allocation to trading
- They predict that the next area to be emphasized by the industry will be dynamic stochastic optimization modelling of strategic problems in investment and asset liability management

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

13

Systematic Portfolio Construction Overview

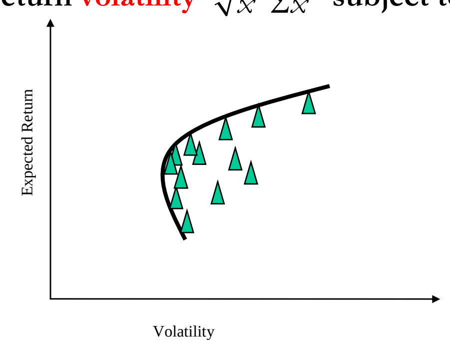
© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

14

Markowitz Portfolio Construction

- Classical mean variance portfolio optimization (MVO) assumes r is distributed multivariate Gaussian (normal) $N(\mu,\Sigma)$ with mean μ and n x n covariance matrix Σ to minimize a tradeoff between expected return and portfolio return volatility $\sqrt{x'\Sigma x}$ subject to constraints Markowitz (1952)

$$\min \quad \mu' x - \lambda x' \Sigma x$$
$$s.t. \quad Ax = b$$



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Departures From Classical Assumptions

- In practice when Σ is estimated as $\hat{\Sigma}$ from historical returns series on individual instruments/strategies the estimate is generally not positive definite and must be 'corrected'
- Extensive modern research on various classes of instruments has shown that approximate normality depends upon
  - the instrument class
  - the period over which returns – assumed independent – are defined (predicted)

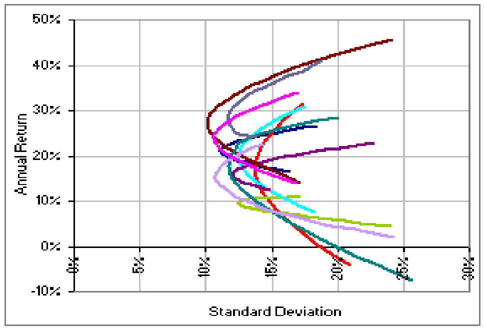
© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

- As a result many alternative distributions for returns have been applied to portfolio construction for stocks, bonds, FX, futures, etc.
- Four characteristics are at issue
  - Higher moments than the two – mean μ and variance σ^2 – which define all moments of the Gaussian distribution
  - Tail behaviour
  - More general dependency between contemporaneous returns than correlation such as that specified by copulas
  - Intertemporal dependencies between returns

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

17

MVO Sensitivity to Input Variations



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

18

Correcting Estimation Error

- For uncorrelated returns the condition number of the estimated covariance matrix will be near one and the contours of portfolio variance will be near spherical and portfolios stable – unfortunately not usual
- Three principal approaches to overcome this practical problem are available
  - Shrinkage techniques which move the historical covariance estimate toward a well-behaved covariance matrix
  - Bayesian approaches which combine the investor's prior views with sample estimates using Bayes theorem   Black & Litterman (1992)
  - Randomization techniques using bootstrap historical resampling or simulation   Michaud (1997)

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

19

Dynamic Optimization

- The uses of MVO or other static optimization portfolio construction techniques depends on the tactical or trading application
- Having briefly surveyed basic MVO and its extensions we will apply scenario based dynamic portfolio construction techniques to strategic institutional and individual asset liability management applications
- But first we will see that such scenario based techniques have randomization advantages over alternatives even in static tactical applications

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

20

Portfolios With Skewed Return Distributions

- For portfolios of instruments – such as bonds – with highly skewed return distributions portfolio volatility is inappropriate as a portfolio risk measure
- More appropriate is a coherent tail based risk measure such as expected shortfall
- However for such risk measures the analytical tractability of the volatility of Gaussian returns is lost and multivariate return distributions must be discretized to result in a finite number of scenarios for computational tractability
- This results in the problem of controlling discrete sampling error for portfolio decisions and returns by sample size and variance reduction

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Scenarios as Randomization Techniques

- Unlike MVO based on Gaussian return distributions scenario based techniques can allow any multivariate return distributions using scenarios that can be simulated, resampled from historical data or generated by quasi-random sequences
- Often such return simulations will involve sampling marginal (individual instrument) returns combined through a specified copula
- In general the scenario approach is a natural and intuitive form of randomization which makes the resulting portfolio decisions robust against a wide range of realized returns

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Long-Short Equity Portfolios for Alpha

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Long-Short Equity Modelling

- The simplest approach to the construction of long-short equity portfolios is to drop the non-negativity condition $x \geq 0$ on portfolio weights in MVO but accounting for the long and short portfolios separately allows control of the hedging ratio
- Hedging ratios of of 120:20 or 150:50 are often used to ensure approximate market neutrality
- The aim of a long-short equity hedging strategy is to have portfolio excess returns uncorrelated with the market (index) in order to generate alpha – or absolute return – in all market conditions
- To this end an additional market-neutral constraint in terms of the CAPM is imposed on MVO where estimates of the betas are used in the actual constraint

$$\hat{\beta}_p := \sum_{i=1}^{n} \hat{\beta}_i x_i = 0$$

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Long/Short Equity Strategy

- Long/short equity fund (hedging ratio as parameter)
- Select long and short portfolios on weekly basis
- Control turnover
- Obey sector constraints
- Stock selection based on historical data *and* analyst opinions
- Used *STOCHASTICS*™ to generate and solve this stochastic programming model
  - StochSim to simulate equity returns
  - GSPL model(s) formulation (selected by user)
  - StochGen / StochOpt to create model instance and select optimal portfolios
  - Proprietary 'trading' interface to generate parameters for simulation, to select model and time windows and display optimization results

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Long/Short Equity Fund System

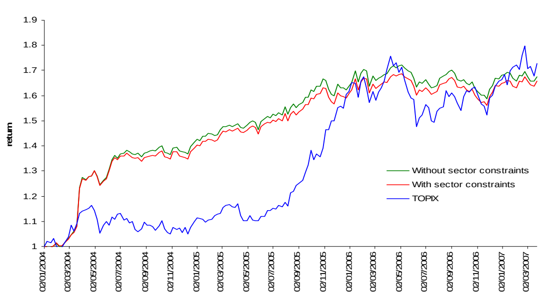
Input | **Optimization** | **Output**

Portfolio Selection
StochGen StochOpt

Simulation
StochSim

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Single Period Model Alpha 2004-2007 10.13%



	Without	With
Return (p.a.)	16.31%	15.84%
Volatility (p.a.)	11.31%	11.19%
Sharpe ratio	1.44	1.42
Beta	**0.35**	**0.35**
Mean number of long stocks	61	62
Mean number of short stocks	40	43
Mean turnover (per week)	1.45%	1.47%

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Multi-Period Model Alpha 2004-2007 21.18%

Return (p.a.)	22.77%
Volatility (p.a.)	12.80%
Sharpe ratio	1.78
Beta	**0.09**
Mean number of long stocks	50
Mean number of short stocks	40
Mean turnover (per week)	7.07%

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Institutional Asset Liability Management

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

29

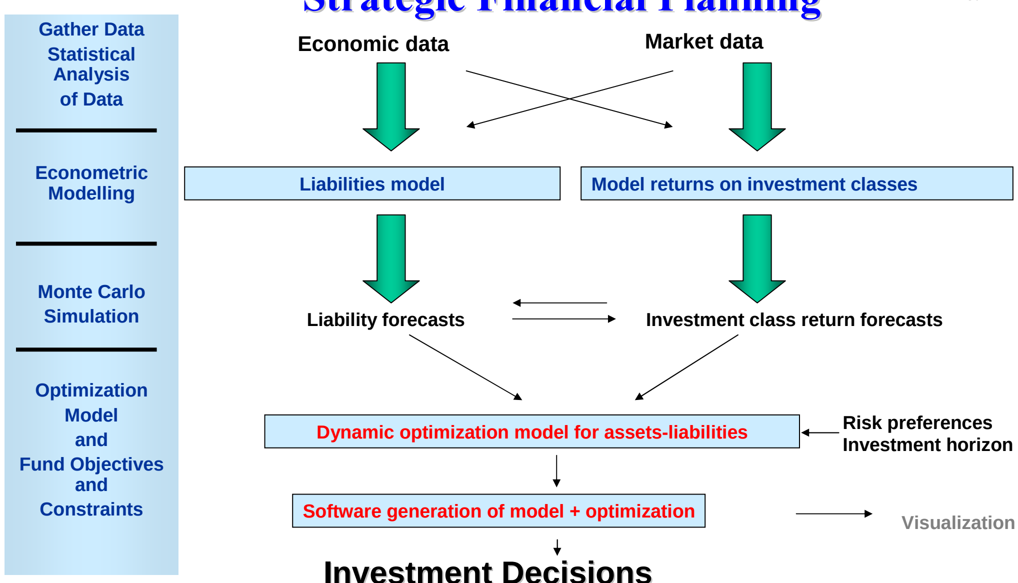
Scenario Based Dynamic Models

- By generating scenarios – by simulation, resampling or sampling using quasi-random sequences – for vector valued processes rather than simply for random vectors the same advantages as for tactical – one period – portfolio construction can be enjoyed for multiperiod strategic portfolio construction
- The techniques used for this setting involve what the actuaries call dynamic financial analysis (simulation) plus simultaneously optimizing decisions using stochastic optimization
- Ability to perform optimal dynamic asset liability management over very long term horizons

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

30

Strategic Financial Planning



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Multi-stage Dynamic Stochastic Programme

$$\min_{x_{t_{1,0}},\dots,x_{t_{1,u}}} f_1(x^{t_{1,u}}) + \mathbb{E}_{\boldsymbol{\omega}^{2,0}}\left\{ \min_{x_{t_{2,0}},\dots,x_{t_{2,u}}} f_2(\boldsymbol{\omega}^{t_{2,0}}, \mathbf{x}^{t_{2,u}}) + \dots + \mathbb{E}_{\boldsymbol{\omega}^{t_{T,0}}|\boldsymbol{\omega}^{t_{T-1,0}}}\left[\min_{x_{t_{T,0}},\dots,x_{t_{T,u}}} f_T(\boldsymbol{\omega}^{t_{T,0}}, \mathbf{x}^{t_{T,0}})\right]\dots\right\}$$

$$\begin{aligned}
&s.t. \\
&A_{11}x_{t_{1,0}} && = b_1 \\
&A_{21}(\boldsymbol{\omega}^{t_{1,1}})x_{t_{0,0}} + A_{22}(\boldsymbol{\omega}^{t_{1,1}})x_{t_{0,1}}(\boldsymbol{\omega}^{t_{1,1}}) && = b_2(\boldsymbol{\omega}^{t_{1,1}}) \quad a.s. \\
&\vdots \\
&A_{Tu+1,1}(\boldsymbol{\omega}^{t_{T+1,0}})x_{t_{1,0}} + \quad \dots \quad + A_{Tu+1,Tu}(\boldsymbol{\omega}^{t_{T+1,0}})x_{t_{T,u}}(\boldsymbol{\omega}^{t_{T+1,0}}) && = b_{Tu+1}(\boldsymbol{\omega}^{t_{T+1,0}}) \quad a.s.
\end{aligned}$$

Deterministic Equivalent

$$\min\left\{ f_1(x^{t_{1,u}}) + \sum_{\Omega_{t_{2,0}}} p_{t_{2,0}}(\omega_{t_{2,0}}) f_{t_{2,0}}(\omega_{t_{2,0}}, x_{t_{2,0}}(\omega_{t_{2,0}}),\dots,x_{t_{2,u}}(\omega_{t_{2,0}})) + \dots + \sum_{\Omega_{t_{T,0}}} p_{t_{T,0}}(\omega_{t_{T,0}}) f_{t_{T,0}}(\omega_{t_{T,0}}, x_{t_{T,0}}(\omega_{t_{T,0}}),\dots,x_{t_{T,u}}(\omega_{t_{T,0}}))\right\}$$

$$\begin{aligned}
&s.t. \\
&A_{11}x_{t_{1,0}} && = b_1 \\
&A_{21}(\omega_{t_{1,1}})x_{t_{1,0}} + A_{22}(\omega_{t_{1,1}})x_{t_{1,1}}(\omega_{t_{1,1}}) && = b_2(\omega_{t_{1,1}}) \quad \omega_{t_{1,1}} \in \Omega_{t_{1,1}} \\
&\vdots \\
&A_{Tu+1,1}(\omega_{t_{T+1,0}})x_{t_{1,0}} + \quad \dots \quad + A_{Tu+1,Tu}(\omega_{t_{T+1,0}})x_{t_{T,u}}(\omega_{t_{T+1,0}}) && = b_{Tu+1}(\omega_{t_{T+1,0}}) \quad \omega_{t_{T+1,0}} \in \Omega_{t_{T+1,0}}
\end{aligned}$$

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Market Asset Returns

Investment securities

- Domestic and International Equities
- Government Bonds
- Corporate Bonds
- Alternatives
- T-bills and all bond coupons
- Treasury Inflation Protected Securities (TIPS)
- Cash
- CPI
- Other fixed assets

Fundamental financial models

Multi-dimensional GBM

$$d\ln X_{i,t} = \mu_i dt + \sigma_i dW_{i,t}$$

Geometric Ornstein Uhlenbeck (OU) process

$$d\ln r_t = (\alpha - \beta\ln r_t)dt + \sigma dW_t$$

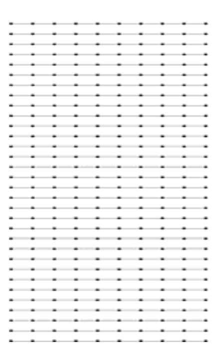
OU process

$$dr_t = (\alpha - \beta r_t)dt + \sigma dW_t$$

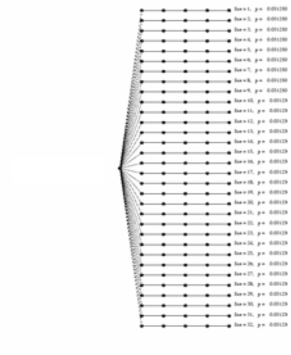
© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Scenario Generation

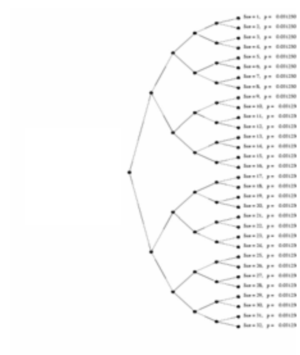
Alternative representations of possible futures



Distribution Problem/DFA



2 Stage Problem

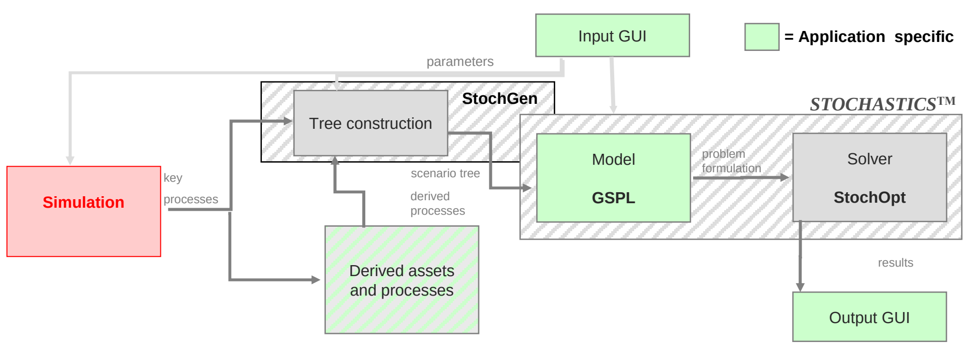


Multistage Problem

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Dynamic Stochastic Programme Implementation

- Simulation is crucial in the optimization process but
  - difficult and complex for any application
  - a separate problem to model building
  - needs to concentrate only on key processes (others can be derived)



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Asset Liability Management

Institutional investors

Problem involves

- Understanding the social security system and pension regulations
- Modelling aggregated liabilities of pension schemes and insurance funds
  - e.g. minimum guaranteed fund returns, insurance claims, corporate pension payments, etc
- Modelling of fundamental economic factors and market returns
  - e.g. inflation and wages, yield curves, asset returns, etc
- Actuarial modelling of mortality, benefit payments to workers, etc
- Corporate decisions regarding funding ratios
- Optimization of contribution rates for employers and employees

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Closed-End Guaranteed Return DC Fund

- After initial cash outlay no contributions are allowed
- Liabilities: nominal or index-linked guarantees
  - Nominal guarantee: Fixed percentage of the initial wealth is guaranteed at a specified date
  - Inflation- or other capital market index linked guarantees
- Assets: EU bonds with maturity 1, 2, 3, 4, 5, 10 and 30 years and the Eurostoxx 50 index
- At the decision times the zero coupon yield with maturity T is a proxy for the fixed coupon rate of a coupon-bearing bond with maturity T
- ALM formulation: Given a set of assets, a fixed planning horizon and set of rebalance dates, find the trading strategy that maximizes the wealth of the fund and minimizes the shortfall below the guaranteed return

$$\max_{\left\{\begin{array}{l}\text{portfolio rebalancing decisions:}\\ a\in A,\omega\in\Omega,t\in T^d\cup\{T\}\end{array}\right\}} E\left\{\alpha\left(\text{wealth}\right)-\beta\left(\text{shortfall}\right)\right\}$$

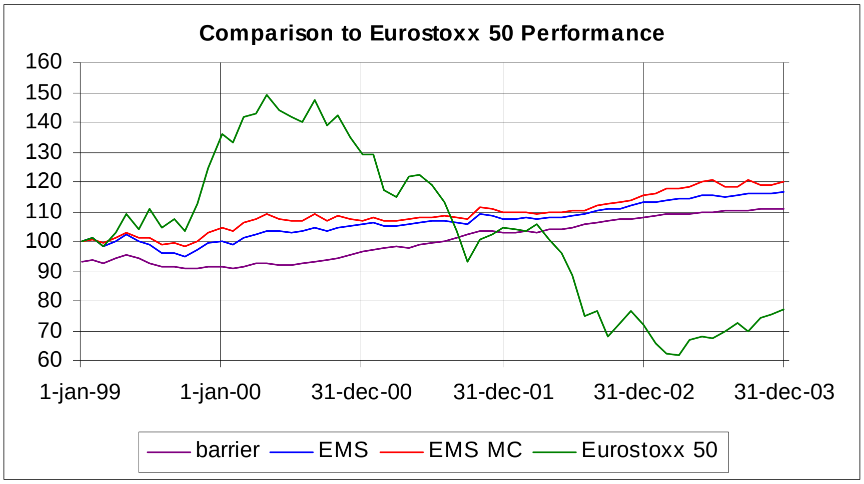
subject to specific constraints

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

37

Historical Backtest 1999-2004

Expected maximum shortfall with monthly checking using the 512.2.2.2 Tree

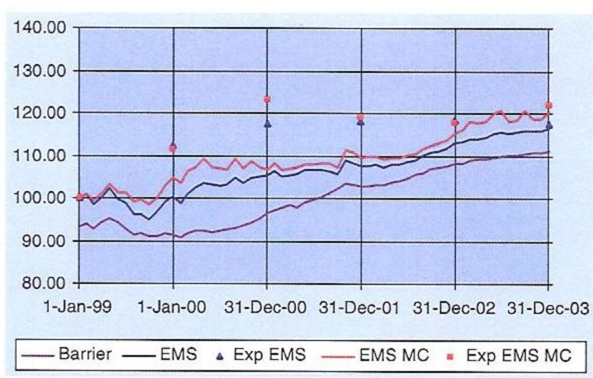


© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

38

Model Predictions and Historical Performance

Expected maximum shortfall for the 512.2.2.2.2 tree



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Portfolio Allocation

Expected maximum shortfall with monthly checking using the 512.2.2.2.2 tree

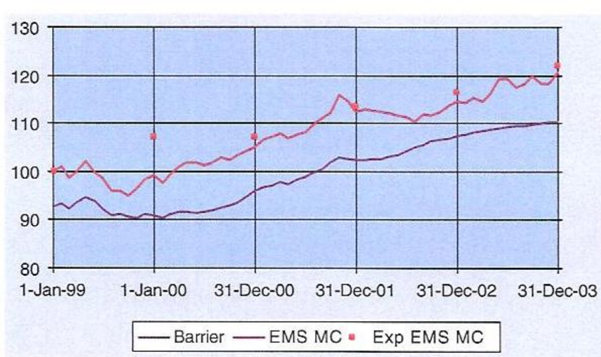
	1y	2y	3y	4y	5y	10y	30y	Stock
Jan 99	0	0	0	0	**0.69**	**0.13**	0	**0.18**
Jan 00	0	0	0	0	**0.63**	0	0	**0.37**
Jan 01	0	0	0	0	**0.37**	**0.44**	0	**0.19**
Jan 02	0	0	0	0	**0.90**	0	0	**0.10**
Jan 03	0	0	**0.05**	0	**0.94**	0	0	**0.01**

Longer bond maturities and smaller bond positions than other versions

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

GBM with Poisson Jumps Equity Index Process

Expected maximum shortfall with monthly checking using the 512.2.2.2.2 tree



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Defined Benefit (DB) Pension Problem

- To improve upon the funding ratio of the pension fund by determining the appropriate investment strategy and contribution rate
  - Multi-objective optimization: (1) maximize wealth subject to (2) minimizing falling short of the target funding ratio while (3) minimizing employer contributions within specified limits
  - Priority of these terms is chosen by setting the parameters in the objective
  - Funding ratio penalty at the horizon
  - Computationally (NP-) hard problem

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Generic Dynamic DB Pension Fund Model

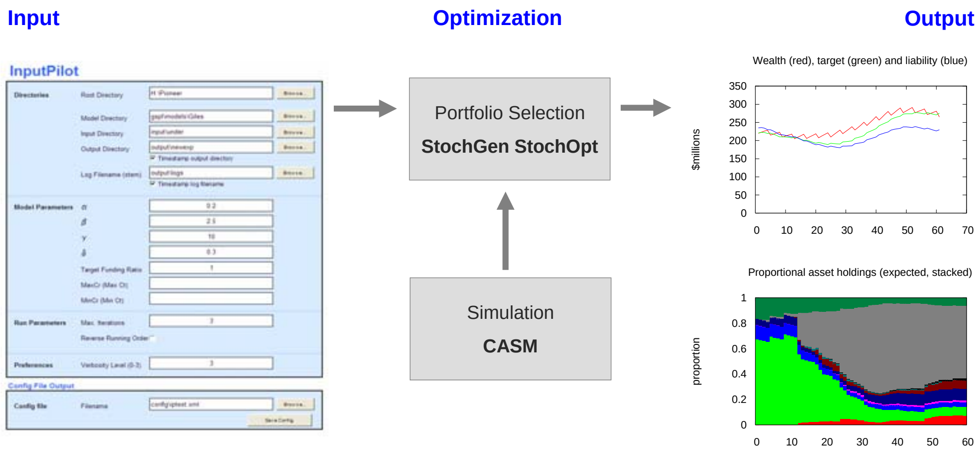
$$\max_{\left\{\substack{\text{portfolio rebalancing decisions:} \\ a \in A,\, \omega \in \Omega,\, t \in T^d \cup \{T\}}\right\}} \left\{\alpha\left(\text{wealth}\right) - \beta\left(\text{shortfall}\right) - \gamma(\text{contributions})\right\}$$

subject to specific constraints

- Specific to pension scheme priorities the model determines optimal positions to meet fund goals:
  - Minimize shortfall of funding target
    - Liabilities are specific to particular pension scheme , e.g. PV of future annuity payments of DB pensions
    - The desired funding ratio F^T and the time horizon T over which F^T is to be achieved determined by fund managers or by legislation
    - Shortfall or surplus of the wealth is defined relative to the target
  - Minimize employer contributions
    - Subject to legal limits and employer wishes
  - Maximize fund wealth
  - Balance between these goals depicts the state of the company and pension trust and their attitude to risk evolving over time
    - a focus on wealth maximization will lead to riskier portfolios than a focus on avoiding shortfall
    - Restriction on the level of contribution will lead to longer time to reach funding target ratio

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

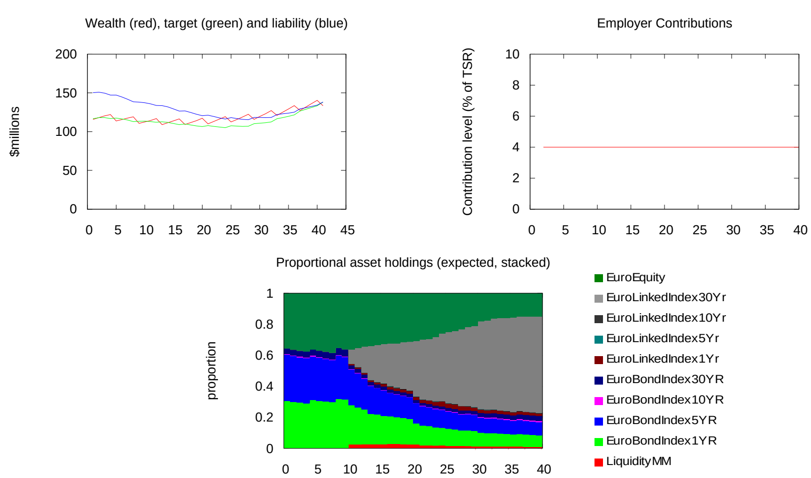
DB Pension Problem



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Under Funded

$F^{initial}$=76%, T=10years, capped contributions at 4%



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Individual Asset Liability Management

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Asset Liability Management
Individual investors: iALM

- *i*ALM generates the life-cycle recommendations for managing wealth regarding selcted (by user) critical decisions along his/her life span such as level of saving or spending at retirement, borrowing, sending children to private schools, buying real estate and so on
- Used *STOCHASTICS*™ with special attention to graphics and computational speed for interactive use

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

*i*ALM Implementation

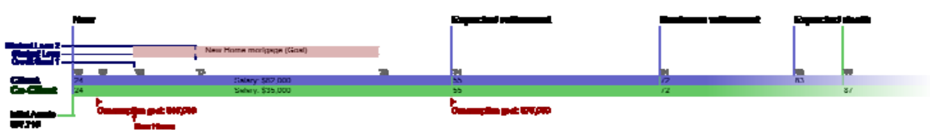
- Modelling random events, e.g. death of individuals, events of sickness, unemployment, etc.
- Modelling of individual's liabilities and incomes *indexed* by inflation
- Accommodate wide range of individual objectives and initial conditions: initial wealth and priorities for all sort of goals
- Consumptions within a range of desirable, acceptable and minimum levels
- Country-specific jurisdictions – taxes, pensions, insurance, mortgages, etc
- Modelling of economic risk factors and market asset returns

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

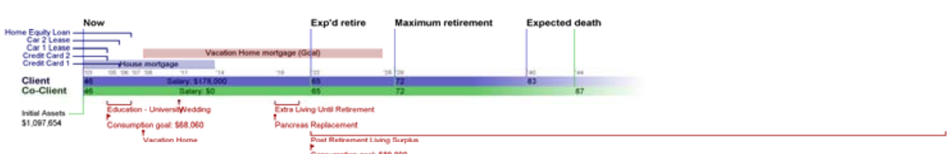
Individual Client Profiles

Variation across objectives/goals, age and time-horizon for investment, liabilities and family structures, income contributions, states of health

Profile_A



Profile_B



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Consumption

- The objective is to maximize the expected value of utility (over all scenarios) of life time consumption in today's currency

$$\mathbb{E}\left[\sum_{t=1}^{T} \mathbf{1}_{\{\text{any alive},t\}} \mathbf{u}_t\right]$$

$$\mathbf{u}_t = \sum_{g \in G} \mathbf{u}_{g,t} - \frac{1}{\boldsymbol{\varphi}_t}\left(\pi^{xs} \mathbf{z}_t^{xs} + \pi^{\tau i} \mathbf{I}_t^{\tau}\right).$$

Here $\mathbf{z}_t^{xs}$ is **excess borrowing**, $\mathbf{I}_t^{\tau}$ is **total tax payment** and $\boldsymbol{\varphi}_t$ is the **inflation index** at t.

- Wealth is generated by the optimal dynamic portfolio allocation given by the
  - implemented optimal current allocation
  - life-time projected optimal dynamic investment policy

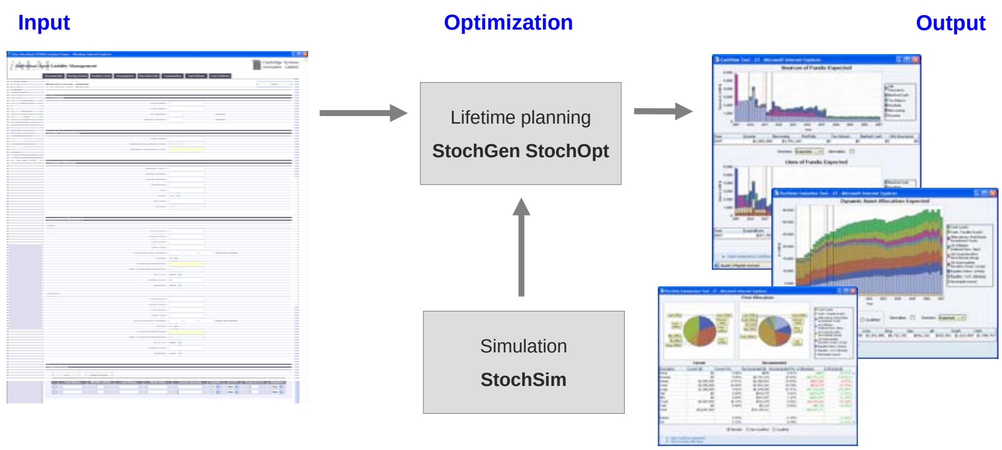
© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

Portfolio Construction

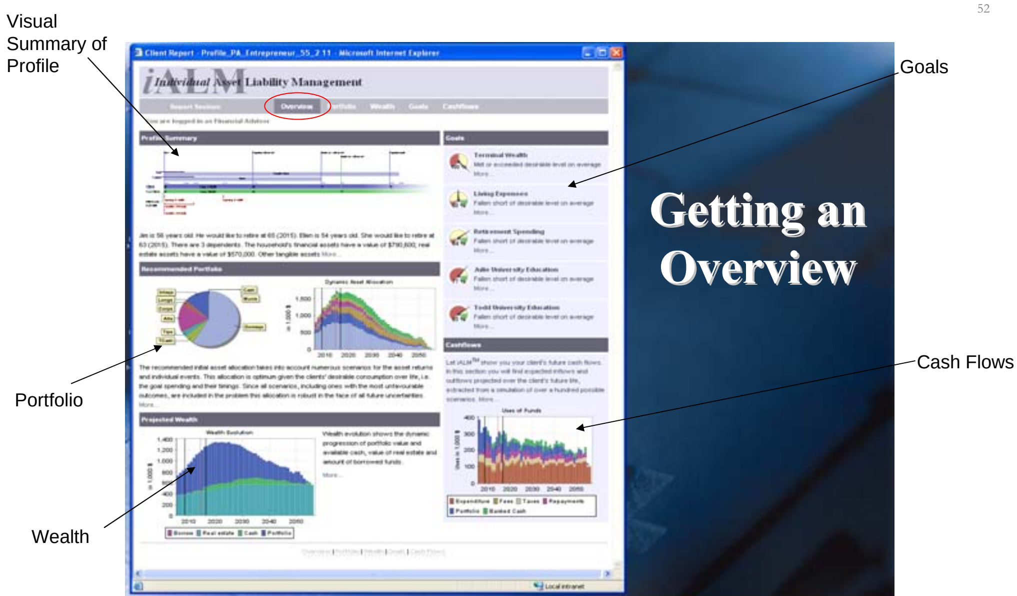
- The optimum portfolio allocations provide life-style enabling returns
- Fundamental constraints
  - Initial holding
  - Portfolio value
  - Portfolio cashflow
  - Asset inventory balance
  - Investment limits, position limits
  - Portfolio drawdown
  - etc

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

iALM



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com



© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

53

Conclusion

- Multistage dynamic ALM models
  - incorporate many future random scenarios
  - generate future decisions by optimizing all decisions simultaneously
  - provide results on future individual scenario evolution
- Risk management is integrated into the process of optimum portfolio construction
- Modelling environment is needed in which any particular problem under uncertainty can be formulated and investigated to dramatically improve the efficiency and effectiveness of DSP for real-world problems
- Visualization provides valuable tools for the analysis of many decision variables over time, individual scenario evolution and aggregates across time and space of financial processes
- Interactive use of financial planning tools leads to a new paradigm in investment management

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com

54

© 2008 Cambridge Systems Associates Limited
www.cambridge-systems.com